

Complex Numbers – Scenario Questions (50)

Representation & Basics ($z = a + bi$, conjugates, operations)

1. An electrical engineer models an alternating current as $z=4+3iz = 4 + 3iz=4+3i$ volts. A second device produces $w=2-iw = 2 - iw=2-i$ volts. Find the combined voltage when both are connected in parallel.
2. A GPS system models a location as $z=-5+7iz = -5 + 7iz=-5+7i$ km east-north. If the base station is at the origin, what are the Cartesian coordinates of this point?
3. A drone is programmed to move in a path given by $z=6-8iz = 6 - 8iz=6-8i$. Represent this displacement as a vector and find its magnitude.
4. A signal has an amplitude represented by $z=3+4iz = 3 + 4iz=3+4i$. Find its conjugate and explain the physical meaning of reflection across the real axis.
5. A ship sails in a direction represented by $z=-2+5iz = -2 + 5iz=-2+5i$. If another sails in the direction $\bar{z}$, explain geometrically how the paths differ.

Modulus & Argument

6. An alternating electrical current is represented by $z=7+24iz = 7 + 24iz=7+24i$. Find its modulus and argument, and explain their physical interpretation.
7. A radar detects an object at point $z=-8+6iz = -8 + 6iz=-8+6i$. Determine the distance and angle of this object relative to the radar origin.
8. A complex impedance in a circuit is given as $z=-3-4iz = -3 - 4iz=-3-4i$. Represent it in modulus-argument (polar) form.
9. A submarine is detected at position $z=12-5iz = 12 - 5iz=12-5i$ km. Find its exact bearing from the control centre.
10. A wind vector is represented as $z=-7+7iz = -7 + 7iz=-7+7i$. Express this in polar form and interpret the direction.

Argand Diagram & Geometry

11. A surveillance drone moves from $z_1=2+3iz_1 = 2 + 3iz_1=2+3i$ to $z_2=-4+iz_2 = -4 + iz_2=-4+i$. Represent the displacement vector on an Argand diagram.
12. A triangle has vertices at $z_1=0z_1 = 0z_1=0$, $z_2=4+3iz_2 = 4 + 3iz_2=4+3i$, $z_3=-2+5iz_3 = -2 + 5iz_3=-2+5i$. Prove it is isosceles.

13. A quadrilateral is formed by the points $z = \pm(3+4i), \pm(3-4i)$. Show that it is a rectangle.
14. A water droplet sensor marks three positions: $z=1+iz = 1 + iz=1+i$, $z=1-iz = 1 - iz=1-i$, $z=-3z = -3z=-3$. Show that they lie on a straight line.
15. A plane radar has three enemy aircraft detected at $z=3+2i, -1+4i, 1-2i$. Show they form a right-angled triangle.

Loci in the Complex Plane

16. A communication tower transmits signals such that the receiving devices must lie on $|z-2|=3$. Describe and sketch this locus.
17. A boat is restricted to sail such that $|z-(1+i)|=|z+(1-i)|$. Show this is the perpendicular bisector of two points.
18. A signal receiver is tuned to pick points satisfying $\text{Re}(z)=3$. What geometric figure does this represent?
19. A drone is allowed to hover such that $|z|\leq 5$. Describe the region and its significance.
20. A lighthouse projects its beam along the locus $\arg(z)=\pi/4$. Represent this locus.

Equations in Complex Numbers

21. A mechanical vibration is represented by $z^2+4z+13=0$. Find the position of equilibrium.
22. A radio signal satisfies $z^2=-16$. Solve for z and interpret.
23. A magnetic resonance scanner records resonance values as solutions to $z^2+6z+25=0$. Find the values.
24. A satellite antenna's orientation is modeled by $z^3=-8$. Find all cube roots and interpret geometrically.
25. A sound wave satisfies the equation $z^4=16$. Find all possible values of z .

De Moivre's Theorem & Trigonometric Identities

26. A turbine rotates with angle θ such that $z = \cos(5\theta) + i\sin(5\theta)$. Express in terms of $\cos\theta, \sin\theta$.
 $z = \cos(5\theta) + i\sin(5\theta)$. Express in terms of $\cos\theta, \sin\theta$.
27. A physics experiment requires evaluation of $(\cos 30^\circ + i\sin 30^\circ)^6$. Simplify using De Moivre's theorem.
28. A windmill rotates through angle θ . Show that
 $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$
 $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$.
29. A pendulum oscillation is represented as $z = (\cos\theta + i\sin\theta)^{10}$. Express in exponential form.
30. A machine crank completes 12 equal rotations. Show that the 12th roots of unity represent its positions.

Roots of Unity & Polynomials

31. A company designs a logo using vertices of a regular hexagon inscribed in a unit circle. Show that these vertices are the 6th roots of unity.
32. A clock's hour positions are represented by the 12th roots of unity. Write them explicitly.
33. A triangular traffic sign is formed by joining every second vertex of an octagon inscribed in a circle. Show this corresponds to cube roots of unity.
34. A robot moves in steps corresponding to the 4th roots of unity. Determine its possible final positions.
35. A decorative star is designed using 7th roots of unity. Write the general formula for their coordinates.

Transformations

36. A transformation sends each complex number z to $w = z + 2$. Describe the effect on a geometric figure in the Argand plane.
37. A reflection of points across the x-axis is given by $w = \bar{z}$. Show the effect on $z = 3 + 4i$.
38. A scaling transformation is defined by $w = 2z$. Show how the locus $|z| = 3$ changes.

39. A rotation by $\pi/3$ about the origin is given by $w = e^{i\pi/3}z$. Find the image of $z = 2 + 2i$.
40. A shear mapping in electronics is modeled by $w = iz$. Describe its effect geometrically.

Applied Problems

41. A phasor in AC circuits is given as $z = 50 \angle 30^\circ$. Convert into rectangular form.
42. A tidal wave displacement is $z = 20(\cos 45^\circ + i \sin 45^\circ)$. Express in rectangular form.
43. A weather balloon drifts with path $z = 5e^{i120^\circ}$. Find its displacement vector.
44. An alternating signal is modeled by $z_1 = 10 \angle 60^\circ$, $z_2 = 15 \angle 120^\circ$. Find the resultant.
45. A submarine sonar records two echoes at $z = 8 + 6i$ and $z = -3 + 4i$. Find the distance between them.

Higher Applications / Proof Style

46. Show that if $|z| = 1$, then $z^{-1} = \bar{z}$. Explain its geometric meaning on the unit circle.
47. Prove that the product of the roots of $z^2 + bz + c = 0$ is c . Give a real-world application in signal balancing.
48. Show that the n th roots of unity form a regular n -gon. Interpret as positions of satellites in equal orbit.
49. Verify that if z_1 and z_2 lie on the circle $|z| = r$, then the perpendicular bisector of the chord z_1z_2 passes through the origin.
50. Prove that the sum of all n -th roots of unity is zero, and explain its symmetry in clock design.

Complex Numbers – Extended Scenario Questions

Representation & Basics

1. A research submarine moves in the path $z = -7 + 2i$. Represent this on the Argand plane and find its real and imaginary displacements.
2. A physics lab measures two oscillations: $z_1 = 2 + 5i$ and $z_2 = -4 + 3i$. Find their sum and difference, and describe the geometric meaning.
3. A cargo drone is programmed to fly to location $z = 9 - 12i$. Find its distance from the control centre at the origin.
4. A software engineer encodes a 2D vector as $z = a + bi$. Show that multiplying by -1 reverses the direction of the vector.
5. A rotating machine is represented by $z = -6 - 8i$. Find the conjugate and explain how it represents a reflection.

Modulus & Argument

6. A ship at position $z = -3 + 3i$ is detected. Find its modulus and argument to determine its distance and bearing.
7. A jet's displacement is modeled by $z = 15(\cos 60^\circ + i \sin 60^\circ)$. Express in rectangular form.
8. A cyclist moves with displacement $z = -4 - 3i$. Find the modulus and argument, giving exact values.
9. A power grid shows impedance $z = 20 + 21i$. Find the magnitude and angle of the impedance.
10. A wind vector is $z = 5\sqrt{3} - 5i$. Find its modulus and argument.

Argand Diagram & Geometry

11. A quadrilateral has vertices at $z = 0, 2, 2 + 2i, 2i$. Show that it is a square.
12. A triangle has vertices $z = 0, 3, 3 + 4i$. Prove it is right-angled.
13. A kite is represented by points $z_1 = 0, z_2 = 4 + 2i, z_3 = 0 + 6i, z_4 = -4 + 2i$. Show its diagonals are perpendicular.
14. A rhombus is drawn with vertices at $z = \pm(2 + i), \pm(2 - i)$. Show that it has equal diagonals.
15. A circle is drawn with centre at origin passing through $z = 6 - 8i$. Find its equation.

Loci in the Complex Plane

16. A drone is restricted to fly along the locus $|z-4|=2$ $|z - 4| = 2$ $|z-4|=2$. Sketch and interpret.
17. A sonar detects signals such that $|z-(2+2i)|=5$ $|z - (2 + 2i)| = 5$ $|z-(2+2i)|=5$. Find the centre and radius.
18. A submarine is restricted to move along $|z+3|=|z-3|$ $|z + 3| = |z - 3|$ $|z+3|=|z-3|$. Find the locus.
19. A region is defined by $|z|<10$ $|z| < 10$ $|z|<10$. What shape is this?
20. A communication path is along $\arg(z-1)=\pi/6$ $\arg(z - 1) = \pi/6$ $\arg(z-1)=\pi/6$. Sketch and interpret.

Equations

21. A computer simulation gives the equation $z^2-2z+5=0$ $z^2 - 2z + 5 = 0$ $z^2-2z+5=0$. Find its solutions.
22. A pendulum oscillation satisfies $z^2+10z+29=0$ $z^2 + 10z + 29 = 0$ $z^2+10z+29=0$. Solve.
23. A drone's path is modeled by $z^3=27$ $z^3 = 27$ $z^3=27$. Find all possible positions.
24. A star design follows $z^4=-16$ $z^4 = -16$ $z^4=-16$. Solve for all complex roots.
25. A satellite orbits at solutions of $z^6=64$ $z^6 = 64$ $z^6=64$. Find all six positions.

De Moivre's Theorem

26. A machine rotates through $(\cos 60^\circ + i\sin 60^\circ)^5$ $(\cos 60^\circ + i\sin 60^\circ)^5$. Simplify.
27. A wind turbine angle is modeled by $z=(\cos \theta + i\sin \theta)^8$ $z = (\cos \theta + i\sin \theta)^8$ $z=(\cos \theta + i\sin \theta)^8$. Write in exponential form.
28. Prove that $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$ $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$ $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$.
29. A generator produces an oscillation represented by $(\cos 15^\circ + i\sin 15^\circ)^{12}$ $(\cos 15^\circ + i\sin 15^\circ)^{12}$ $(\cos 15^\circ + i\sin 15^\circ)^{12}$. Find its value.
30. A polygon inscribed in a unit circle has vertices from the 8th roots of unity. Show these correspond to regular octagon vertices.

Roots of Unity

31. A security system lights up 5 equally spaced sensors on a circle. Show these are the 5th roots of unity.
32. A festival decoration uses points corresponding to 9th roots of unity. Write the coordinates.
33. A regular pentagon is designed on a unit circle. Show its vertices are 5th roots of unity.
34. A factory machine completes 4-step cycles. Represent its stages as 4th roots of unity.
35. A regular dodecagon (12-sided polygon) is formed by 12th roots of unity. Write them.

Transformations

36. A translation is given by $w = z + (3 - 2i)$. Find the image of $z = -1 + 4i$.
37. A rotation of 90° is given by $w = iz$. Show the effect on $z = 5 - 2i$.
38. A scaling by factor 3 is defined by $w = 3z$. Find the image of $|z| = 2$.
39. A reflection across the y-axis is given by $w = -\bar{z}$. Find the image of $z = 4 + 5i$.
40. A transformation is defined by $w = 1/\bar{z}$. Describe its effect on the unit circle.

Applied Problems

41. A sound wave has amplitude $z = 12(\cos 120^\circ + i\sin 120^\circ)$. Convert to rectangular form.
42. A jet engine vibration is modeled by $z = 5\angle 210^\circ$. Express in Cartesian form.
43. A phasor current is given by $z = 30\angle 45^\circ$. Convert into rectangular coordinates.
44. Two alternating signals $z_1 = 40\angle 30^\circ$, $z_2 = 20\angle 150^\circ$ are combined. Find the resultant.
45. A sensor records echoes at $z = -7 + 24i$ and $z = 9 - 12i$. Find their distance.

Higher Applications

46. Prove that for $|z|=1$ $|z| = 1/|z|=1$, the argument of $z^{-1}\bar{z}$ is the negative of the argument of z .
47. Show that the sum of conjugate complex roots is real. Apply this to $z^2+4z+13=0$ $z^2 + 4z + 13 = 0$.
48. Prove that the cube roots of unity form an equilateral triangle.
49. If $|z|=r$ $|z| = r/|z|=r$, show that $|z^{-1}\bar{z}| = r/|z|=r$. Interpret physically.
50. Prove that the diagonals of a square inscribed in a unit circle intersect at the origin, using complex coordinates.